2018

Full Marks - 40

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer ***all*** questions

1. a) What is a sample space ? Describe the sample space of executing an if....then....else... statement 5 times. 1½

   b) Of all the graduate students in a University, 70% are women and 30% are men. Suppose that 20% and 25% of the female and male population, respectively, smoke cigarettes. What is the probability that a randomly selected graduate student is : 2½

   i) A woman who smokes ?

   ii) A man who smokes ?

   iii) A smoker ?

c) Given a list of events, $A_1, A_2, \ldots\ldots, A_n$, show that

$$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1 | (A_2 \cap A_3 \cap \ldots\ldots \cap A_n)) \cdot P(A_2 | (A_3 \cap A_4 \cap \ldots\ldots \cap A_n)) \cdot \vdots \cdot P(A_{n-1} | A_n) \cdot P(A_n).$$ 4

d) Prove that $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$, where A and B are independent. 2

OR

e) Show that if even B is contained in even A, then $P(A) \geq P(B)$. 1

f) Prove the relation :

$$A \cap (A \cup B) = A$$

by using the axioms of event algebra. 1½

g) If $A_1, A_2, \ldots\ldots, A_n$ are any events, then prove that

$$P\left(\bigcup_{i=1} A_i\right) = \sum_i P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \ldots\ldots$$

$$+(-1)^{n-1} P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n)$$ 2½

h) A certain firm has plants A, B and C producing respectively 35%, 15% and 50% of the total output. The probabilities of a non-defective product are, respectively 0.75, 0.95 and 0.85. A customer receives a defective product. What is the probability that it came from plant C ? 5

2. a) Discuss Poisson probability mass function. Show that the function is satisfying the properties of pmf. 5

b) Consider a normalized floating point number in base β so that the mantissa, X, satisfies the

condition $1/\beta \leq X < L$. Experience shows that X has the reciprocal density :

$$f_X(x) = \frac{K}{x}, K > 0.$$

Determine :

i) The value of K.

ii) The distribution function of X. 5

OR

c) i) Discuss exponential density function. 2

ii) Consider a University Computer Centre with an average rate of Job Submission X = 0.1 jobs per second. Assuming that the number of arrivals per unit time is Poisson distributed, the inter-arrival time, X is exponentially distributed with parameter λ. What is the probability that an interval of 10 seconds elapses without job submission. 3

d) Discuss Binomial pmf and normal density function. 5

3. a) The number of hardware failure of a computer system in a week of operation has the following pmf :

No. of failures	0	1	2	3	4	5	6
Probability	.18	.28	.25	.18	.06	.04	.01

i) Find the expected number of failures in a week.

ii) Find the variance of the number of failures in a week. 3

b) State central limit theorem. 2

c) Prove that $E[\bar{X}] = \mu$. 5

OR

d) Consider discrete random variables X and Y with the joint pmf as shown below : 5

		Y		
		−1	0	1
X	−2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
	−1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$
	1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$
	2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

Are X and Y independent ? Are they correlated ?

e) Show that $Cov^2(X, Y) \leq Var[X]\ Var[Y]$. 5

4. a) Derive the equation of the line of regression of X on Y as : 5

$$x - \bar{x} = \int \frac{\sigma_y}{\sigma_x}(y - \bar{y})$$

b) Write a program to find out the correlation between two variables X and Y. 5

OR

c) Write a program to implement the least square curve fitting for the given data. 5

d) Compute the correlation coefficient from the below given data : 5

X	100	120	135	135	115	110	120
Y	50	40	60	80	80	55	65

□□