2017

Full Marks - 40

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer ***all*** questions

1. a) If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10) with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. 5

   b) Prove that

   $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$. 5

   OR

   c) A lot of transistors contains 0.6 percent defectives. Each transistor is subjected to a test that correctly identifies a defective but also misidentifies as defective about two in every 100 good transistors. Given that a randomly chosen transistor is declared defective by the tester, compute the probability that it is actually defective. 6

d) State and prove Bayes Rule. State an application area in which this rule is widely used. 4

2. a) Enumerate the properties of probability mass function. Show that Binomial probability mass function is satisfying all properties of pmf. 5

b) The probability of error in the transmission of a bit over a communication channel is $p = 10^{-4}$. What is the probability of more than three errors in transmitting a block of 1000 bits ? 5

OR

c) Consider a random variable X defined by the CDF :

$$F_x(x) = \begin{cases} 0 & \text{if } x<0 \\ \frac{\sqrt{x}}{2} + \frac{1-e^{-\sqrt{x}}}{2} & \text{if } 0 \le x \le 1 \\ \frac{1}{2} + \left(1-e^{-\sqrt{x}}\right)/2 & \text{if } x>1 \end{cases}$$

Show that this function satisfies all properties of CDF. 6

d) Discuss Markov property of exponential distribution. 4

3. a) If X and Y are two random variables then show that the expectation of their sum is the sum of their expectation. 5

b) Show that the PGF of a poisson random variable is $e^{-\lambda(1-z)}$. 5

OR

c) If X and Y are independent random variable, then show that

$$var[X+Y] = var[X] + var[Y]. \quad 2$$

d) Random variables X and Y are said to be orthogonal iff $E[XY] = 0$. 8

i) If X and Y are orthogonal determine the conditions under which they are uncorrelated.

ii) If X and Y are uncorrelated, determine the conditions under which they are orthogonal.

4. a) Derive the equation of the line of regression of X on Y as

$$X - \bar{X} = \rho \frac{\sigma_Y}{\sigma_X}\left(Y - \bar{Y}\right). \quad 4$$

b) Write a program in any high level programming language to find out the Karl Pearsons correlation coefficient between two random variables. 6

OR

c) Write a program in any high level programming language to generate a random number using congruence modulo method. 5

d) Write steps of problem solving approach of regression through least square line fitting. 5

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