

2019

Full Marks - 40

Time - 3 hours

The questions are of equal value.

Answer *all* questions

1. Write brief answers on the following :

- a) What is equivalence relation ? Explain with an example.
- b) Define Pigeonhole Principle and explain with an example.
- c) Define Euler graph and Hamilton graph.
- d) What are different types of FA exist with memory ?

2. a) Using mathematical induction prove that

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} \leq 2 - \frac{1}{n}, \forall n \in \mathbb{N}$$

[2]

- b) Prove that if an integer a for which $(a - 2)$ is divisible by 3 then $a^2 - 1$ is divisible by 3.

OR

- c) Let $A = \{1, 2, 3, \dots, 7\}$ and $R = \{(x, y) | (x - y) \text{ is divisible by } 3\}$ in A . Show that R is an equivalence relation.
- d) Find out the transitive closure of R of $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)\}$.

3. a) Find the number of ways in which 8 men and 4 women stand in a circle such that two women are next to each other.

- b) State and prove method of inclusion and exclusion.

OR

Solve the recurrence relation

$$a_n = 7a_{n-2} + 6a_{n-3}, \quad a_1 = 3, \quad a_2 = 6, \quad a_3 = 10.$$

What is the degree of this linear homogeneous relation ?

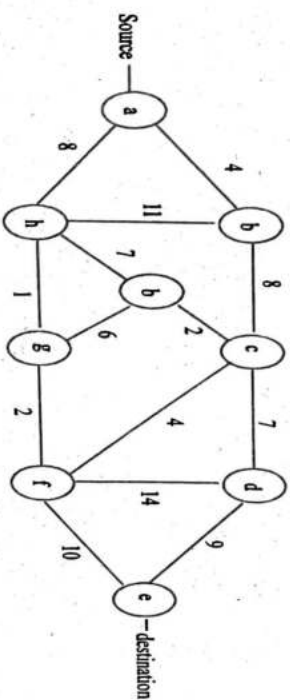
[3]

4. a) Prove that a simple non-directed graph G is a tree iff G is connected and contains no cycle and also prove there is atleast one vertex with degree 1.

- b) Let the number of edges of a graph be m and number of vertices be n , then show that G has a Hamiltonian circuit if $n \geq \frac{1}{2}(n^2 - 3n + 6)$

OR

Write Prim's algorithm and implement it to find MST for the given problem



5. a) Construct NFA for the regular expression $(0+1)^*(00+11)(0+1)^*$ and convert it into DFA.

[4]

- b) Construct a DFA which accept set of all string divisible by 5 where $\Sigma = \{0, 1\}$.

OR

- c) Design a PDA for $L = \{a^n ba^n \mid n \geq 1\}$
- d) Design a TM over $\Sigma = \{1, b\}$ to accept the language $L = \{ww^R \mid w \in (a, b)^+\}$.

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