2018

Full Marks - 40

Time - 3 hours

The questions are of equal value

Answer ***all*** questions

1. Using Mathematical Induction prove that

$$1^2 - 2^2 + 4^2 \ldots\ldots (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}. \quad 10$$

OR

Define equivalence relation. Let A = {1, 2, 3, ... 7} and R = {(x, y) | (x – y) is divisible by 3} in A. Show that R is an equivalence relation. 10

2. Solve the recurrence relation

$$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0 \text{ for } n \geq 3. \quad 10$$

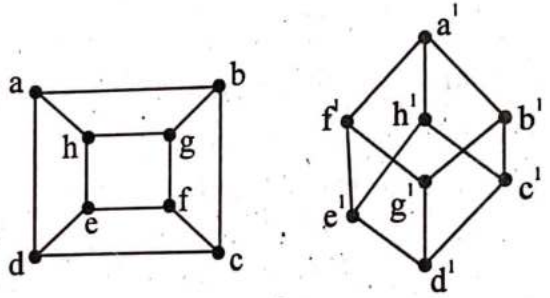
OR

a) Find the total number of Positive integers that can be formed from the digits 1, 2, 3, 4, and 5 if no digit is repeated in any one integer. 5

b) Solve the recurrence relation for a particular solution $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = n2^n$. 5

3. a) For any simple graph G, prove that the number of edges of G is less than or equal to $\frac{1}{2n(n-1)}$ where n is the number of vertices of G. 5

b) What is graph isomorphic. Check the given graph are isomorphic or not. 5



OR

c) Show that if G is a polyhedral graph, then there is a region of degree ≤ 5. 5

d) Prove that a connected graph is a tree if and only if each edge is a cut edge. 5

4. a) Construct an NFA for regular expression $(0 + 1)^* (00 + 11) (0 + 1)^*$ and convert it to its DFA form. 5

b) What is pumping Lemma. Prove that $L = \{0^n \mid n \text{ is prime}\}$ is not regular. 5

OR

c) Construct a PDA which accept the Language $L = \{0^n 1^n 0^{m+n} \mid n \geq 1\}$ 5

d) Design a Turing Machine over $\Sigma = \{a, b\}$ to accept $L = \{WW^R \mid w \in (a, b)^+\}$ 5

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